Final Exam Statistical Signal Processing, 2014-2015

Lecturer: Prof. Dr. Saleem Zaroubi
Teaching assistant: Mr. Sarrvesh Sridhar

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The exam duration is 3 hours ($14^{00} - 17^{00}$).
The number of points given to each question is indicated next to it. The grade will be based on your answers to all questions.
Indicate clearly the steps in your solution and provide sufficient text.

Question I.

Let $\{x_1, x_2, \dots, x_N\}$ be i.i.d. measurements drawn from a Poisson distribution

$$p(x_i;\theta) = \frac{\theta^{x_i}}{x_i!}\exp(-\theta). \tag{1}$$

Where θ is a positive real number that is equally to the mean and variance of the distribution.

1. **Write the expression for the Likelihood function, $p(x_1, x_2, \dots, x_N|\theta)$, for this problem. and calculate the maximum likelihood estimator for θ.**
2. **Show that the ML estimator is unbiased.**

   Now assume that the parameter is a random variable with a *prior* of an exponential distribution,

$$p(\theta) = \begin{cases} \lambda e^{-\lambda\theta} & \text{if } \theta > 0; \\ 0 & \text{otherwise.} \end{cases}$$

   Where λ is a given constant.

3. **What is the *posterior* pdf $p(\theta|x_1, x_2, \dots, x_N; \lambda)$?**
4. **Find the MAP estimator of θ and discuss its difference with respect to the ML solution.**

(34 points)

Question II.

In a certain experiment the outcome is measured by x, which is either 1 or 0 depending on the success or failure of the trials, respectively. Let $\{x_1, x_2, \ldots, x_N\}$ denote the outcome of N independent such experiments, namely, each x_i is drawn from an *i.i.d.*. Assume that $X = x_1 + x_2 + \cdots + x_N$ is the total number of successes in all trials.

S, the probability of success at any given trial, is an unknown parameter. The pdf for each trial is with outcome x_i follows a Bernoulli distribution,

$$p(x_i|S) = \begin{cases} S & \text{if } x_i = 1; \\ 1 - S & \text{if } x_i = 0. \end{cases}$$

Notice that Bernoulli distribution could also be written as $p(x_i|S) = S^{x_i}(1-S)^{1-x_i}$ for $x_i \in 0, 1$.

1. **If you know that the number of success after N trials is X. Using these two quantities, define a simple estimator for S (the probability for success).**
2. **Show that such an estimator is unbiased.**
3. **Find the variance of this estimator.**
4. **Write an expression for the joint probability and show that your estimator attains the CRLB.**

(33 points)

Question III.

A number of short questions. Each with 11 points.

1. **What is the relationship between SVD and the LSE of Linear models? Suggest how one regularise the LSE solution for the smallest singular values.**
2. **A data vector of order N that has only white noise, namely, $x_i \sim \mathcal{N}(0, \sigma^2)$ and the data points are *i.i.d*s. Show that the power spectrum of such a (white Gaussian noise) vector is flat.**
3. **Explain briefly the concept of Wiener filtering and its different aspects.**

(33 points)

Good Luck!